Liquidity and Portfolio Choice: A Unified Approach

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Liquidity as a shadow allocation

We treat liquidity as a shadow allocation.

If liquidity is deployed to raise expected utility, we attach a shadow asset to tradable assets to capture this incremental benefit.

If liquidity is deployed to prevent a decline in expected utility, we attach a shadow liability to assets that are not tradable.

Liquidity as a shadow allocation

This approach improves upon other methods for incorporating liquidity into portfolio choice in four fundamental ways:

1. It mirrors what actually occurs within a portfolio
2. It maps units of liquidity onto units of return and risk, so that investors can analyze liquidity within the same context as other portfolio decisions.
3. It distinguishes absolute illiquidity from partial illiquidity and enables investors to treat these attributes within a single unifying framework.
4. It recognizes that liquidity serves not only to meet demands for capital, but to exploit trading opportunities as well, thereby revealing that investors bear an illiquidity cost to the extent any fraction of a portfolio is immobile.

Liquidity as a shadow allocation

It is important to distinguish between absolute and partial illiquidity

- Absolute illiquidity refers to situations in which the investor is contractually proscribed from trading the asset, or the cost of trading it is prohibitively expensive.

  We do not attach a shadow asset to assets that are absolutely illiquid, but in some cases we attach a shadow liability to them.

- Partial illiquidity refers to situations in which it is costly to trade or investors can trade only with delay, and these costs and delays vary across assets and through time.

  We sometimes attach a shadow asset to assets that are partially illiquid, but we reduce its expected return by the cost of trading or by the extent to which trading delays limit the opportunity to benefit from trading.

How investors benefit from liquidity

- **Rebalancing**
- **Capital calls**
- **Market timing skill**
- **New opportunities**
- **Shifting risk appetite**

Even though these liquidity benefits are driven by different purposes, we can measure all of them in units of expected return and risk.

Portfolio construction

- We consider a two asset portfolio, split evenly between liquid equities and bonds.
- Based on assumptions for expected return, risk, correlation, and risk aversion, we can determine the required return of equities necessary to justify a 50% allocation.
- At this stage, we do NOT yet consider the impact of the shadow liquidity asset.
- We can confirm that the portfolio weights are optimal when the marginal utility of increasing the equity allocation is equal to the marginal utility of increasing the bond allocation.

Analytical construct

$$E(U) = r_e w_e + r_b w_b - \lambda(\sigma_e^2 w_e^2 + \sigma_b^2 w_b^2 + 2\rho\sigma_e\sigma_b w_e w_b)$$

$$\frac{\partial U}{\partial w_e} = r_e - \lambda(2\sigma_e^2 w_e + 2\rho\sigma_e\sigma_b w_b)$$

$$\frac{\partial U}{\partial w_b} = r_b - \lambda(2\sigma_b^2 w_b + 2\rho\sigma_e\sigma_b w_e)$$

E(U) = expected utility
r_e = expected equity return
r_b = expected bond return
σ_e = equity standard deviation
σ_b = bond standard deviation
w_e = equity weight
w_b = bond weight
λ = coefficient of risk aversion
ρ = correlation of equity and bonds

Numerical example

Required return for equities					
	Liquid equity				
	Starting assumptions				
Required equity return	**8.75%**				
Bond return	5.00%				
Equity volatility	20.00%				
Bond volatility	5.00%				
Equity/bond correlation	0.5000				
Risk aversion	1				
Equity weight	50%				
Bond weight	50%				
Marginal utility equities	0.04250				
Marginal utility bonds	0.04250				
Derivative difference	0.0000				

Introducing an illiquid asset

- Next, we substitute illiquid equity for liquid equity.
- The volatility and correlation of the equity asset class will change, and we can calculate the new return required to justify a 50% allocation.
- We still do NOT yet consider the impact of the shadow liquidity asset.

Numerical example

Required return for equities					
	Liquid equity	Illiquid equity			
	Starting assumptions	Substitute illiquid equity			
Required equity return	**8.75%**	**5.31%**			
Bond return	5.00%	5.00%			
Equity volatility	20.00%	**7.50%**			
Bond volatility	5.00%	5.00%			
Equity/bond correlation	0.5000	**0.2500**			
Risk aversion	1	1			
Equity weight	50%	50%			
Bond weight	50%	50%			
Marginal utility equities	0.04250	0.04656			
Marginal utility bonds	0.04250	0.04656			
Derivative difference	0.0000	0.0000			

Performance fees and volatility

- Performance fees underestimate a fund's downside volatility, because they truncate the upside but not the downside.
- We, therefore, correct for this downward bias when measuring volatility.

Analytical construct

$$r_n = r_g - b - \max\left(0, p \cdot (r_g - b)\right)$$

$$r_g = \begin{cases} r_n + b & for\ r_n < 0 \\ \dfrac{r_n}{1-p} & for\ r_n \geq 0 \end{cases}$$

r_n = return net of fees
r_g = return gross of fees
b = base fee
p=performance fee

Numerical example

Required return for equities					
	Liquid equity	Illiquid equity	Illiquid equity		
	Starting assumptions	Substitute illiquid equity	Correct for fees		
Required equity return	**8.75%**	**5.31%**	**5.75%**		
Bond return	5.00%	5.00%	5.00%		
Equity volatility	20.00%	7.50%	**10.00%**		
Bond volatility	5.00%	5.00%	5.00%		
Equity/bond correlation	0.5000	0.2500	**0.3000**		
Risk aversion	1	1	1		
Equity weight	50%	50%	50%		
Bond weight	50%	50%	50%		
Marginal utility equities	0.04250	0.04656	0.04600		
Marginal utility bonds	0.04250	0.04656	0.04600		
Derivative difference	0.0000	0.0000	0.0000		

Price smoothing and volatility

- Appraisal and fair value pricing may introduce smoothing which dampens observed volatility.
- We de-smooth the illiquid asset returns to offset the reduction in the observed standard deviation introduced by appraisals and fair value pricing.
- We use a first order auto-regressive model using least squares.

Analytical construct

$$r_t = A_0 + A_1 r_{t-1} + \varepsilon$$

$$r_t' = \frac{r_t - A_1 r_{t-1}}{1 - A_1}$$

r_t' = de-smoothed return observation at time t
r_t = return observation at time t
A_0 = intercept
A_1 = regression coefficient
ε = error term

Numerical example

Required return for equities					
	Liquid equity	Illiquid equity	Illiquid equity	Illiquid equity	
	Starting assumptions	Substitute illiquid equity	Correct for fees	Correct for smoothing	
Required equity return	**8.75%**	**5.31%**	**5.75%**	**13.75%**	
Bond return	5.00%	5.00%	5.00%	5.00%	
Equity volatility	20.00%	7.50%	10.00%	**30.00%**	
Bond volatility	5.00%	5.00%	5.00%	5.00%	
Equity/bond correlation	0.5000	0.2500	0.3000	**0.5000**	
Risk aversion	1	1	1	1	
Equity weight	50%	50%	50%	50%	
Bond weight	50%	50%	50%	50%	
Marginal utility equities	0.04250	0.04656	0.04600	0.04000	
Marginal utility bonds	0.04250	0.04656	0.04600	0.04000	
Derivative difference	0.0000	0.0000	0.0000	0.0000	

Including the shadow liquidity allocation

- Finally, we adjust the expected return, standard deviation, and correlation of liquid bonds to account for the presence of the shadow liquidity asset.
- Note that in practice, rather than re-state the liquid assets' return and risk estimates, we can simply attach an overlay which is constrained to the same allocation as the liquid assets.
- For illustrative purposes, we consider a shadow asset but no shadow liability in this example.

Analytical construct

$$\mu_{bl} = \mu_b + \mu_l$$

$$\sigma_{bl}^2 = \sigma_b^2 + \sigma_l^2$$

$$\rho_{bl,i} = \frac{\rho_{b,i}\sigma_i}{\sigma_{bl}}$$

μ_{bl} = expected return of bonds with shadow liquidity asset
μ_l = expected return of shadow liquidity asset
σ_{bl} = standard deviation of bonds with shadow liquidity asset
σ_l = standard deviation of shadow liquidity asset
$\rho_{bl,i}$ = correlation of bonds (with shadow liquidity asset) and equity

Numerical example

Required return for equities					
	Liquid equity	Illiquid equity	Illiquid equity	Illiquid equity	Illiquid equity
	Starting assumptions	Substitute illiquid equity	Correct for fees	Correct for smoothing	With shadow liquidity asset
Required equity return	**8.75%**	**5.31%**	**5.75%**	**13.75%**	**15.50%**
Bond return	5.00%	5.00%	5.00%	5.00%	**7.00%**
Shadow asset return					2.00%
Equity volatility	20.00%	7.50%	10.00%	30.00%	30.00%
Bond volatility	5.00%	5.00%	5.00%	5.00%	**7.07%**
Shadow asset volatility					5.00%
Equity/bond correlation	0.5000	0.2500	0.3000	0.5000	**0.3536**
Risk aversion	1	1	1	1	1
Equity weight	50%	50%	50%	50%	50%
Bond weight	50%	50%	50%	50%	50%
Marginal utility equities	0.04250	0.04656	0.04600	0.04000	0.05750
Marginal utility bonds	0.04250	0.04656	0.04600	0.04000	0.05750
Derivative difference	0.0000	0.0000	0.0000	0.0000	0.0000

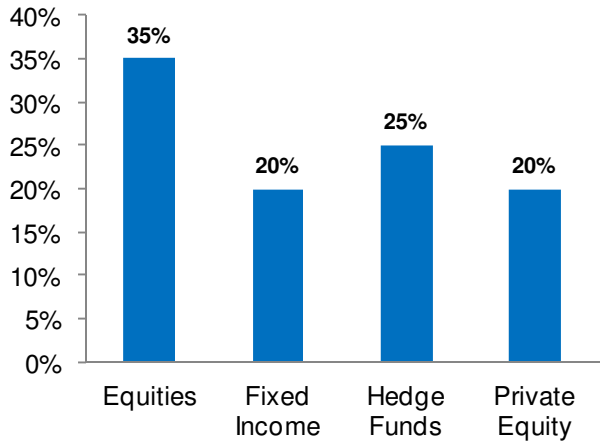
Case study

- We illustrate our approach by applying it to a representative institutional portfolio.
- We employ simulation to quantify the impact of three uses of liquidity: market timing, portfolio rebalancing, and funding capital calls.
- The aggregation of these effects gives us the expected return and risk of the **shadow asset** and the cost and risk of the **shadow liability**.
- We compare optimal portfolio allocations that account for the impact of liquidity with those that ignore it.

Optimization inputs

Asset class expected returns, risk, and correlations						
Asset Class	**Expected Return* (%)**	**Standard Deviation (%)**	**Correlations**			
			Equities	**Fixed Income**	**HF**	**PE**
Equities	9.4	15.0	1.00			
Fixed Income	4.0	8.0	0.25	1.00		
Hedge Funds	7.0	15.0	0.25	0.25	1.00	
Private equity	15.8	30.0	0.75	0.00	0.25	1.00

Optimal portfolio weights
(ignoring liquidity)



* Expected returns are implied assuming a risk aversion of 2.

Market timing

- Some investors are skilled at anticipating the relative performance of asset classes or risk factors.
- The expected return and risk associated with this market timing skill accrue to the **shadow asset**.
- We assume that market timing skill produces an excess return equal to 0.4% with excess risk equal to 0.8%.

Return and risk of the shadow asset and liability

Liquidity benefits and illiquidity penalties			
	Return (bps)	**Risk (bps)**	**Attached to:**
Benefits			
Market timing	40	80	Liquid assets
Total shadow asset	**40**	**80**	**Liquid assets**

Rebalancing: simulation procedure

- Price changes among the component assets of a portfolio cause the actual weights of the portfolio to drift away from the optimal targets, and the portfolio becomes sub-optimal.
- Because liquidity serves to restore expected utility and not to improve it, we attach this cost as a **shadow liability** to the illiquid assets.
- We estimate this cost by simulating 10,000 five-year paths given assumptions about the portfolio allocation as well as the expected return, risk, and correlations for each asset class.
- We consider two scenarios: one in which we rebalance the portfolio to its target weights annually (at a cost of 20 bps) and one in which we do not rebalance the weights.
- We compute the illiquidity penalty as the difference between the certainty equivalents of the portfolios' ending wealth distributions in the two scenarios.

Return and risk of the shadow asset and liability

Liquidity benefits and illiquidity penalties			
	Return (bps)	**Risk (bps)**	**Attached to:**
Benefits			
Market timing	40	80	Liquid assets
Total shadow asset	**40**	**80**	**Liquid assets**
Penalties			
Sub-optimality cost from asset drift	46	0	Illiquid assets

Capital calls

- Investors who are unable to generate sufficient inflows to meet capital calls may need to liquidate part of the portfolio to fund these capital calls, which drives the portfolio away from its optimal mix.
- As with rebalancing, we measure this sub-optimality cost in certainty equivalent units and attach it to the illiquid assets as a **shadow liability**.
- In rare instances investors may be unable to liquidate a sufficient fraction of their portfolios to meet capital calls, thus requiring them to borrow.
- In these instances, it makes sense to attach another **shadow liability** to the illiquid assets to reflect the cost and uncertainty of borrowing.

Capital calls: simulation procedure

- We assume there is a 10% probability the investor will need to raise cash to meet capital calls in a given month.
- As long as cumulative capital calls are below 15% of initial portfolio value, we draw on the liquid assets (proportionally) to meet capital calls.
- When cumulative capital calls exceed 15% of initial portfolio value, we begin borrowing to meet capital calls, at an interest rate of 5%.
- We estimate the sub-optimality cost arising from liquidations as the difference in certainty equivalents.
- We introduce an additional liability to capture the cost and uncertainty of borrowing.

Return and risk of the shadow asset and liability

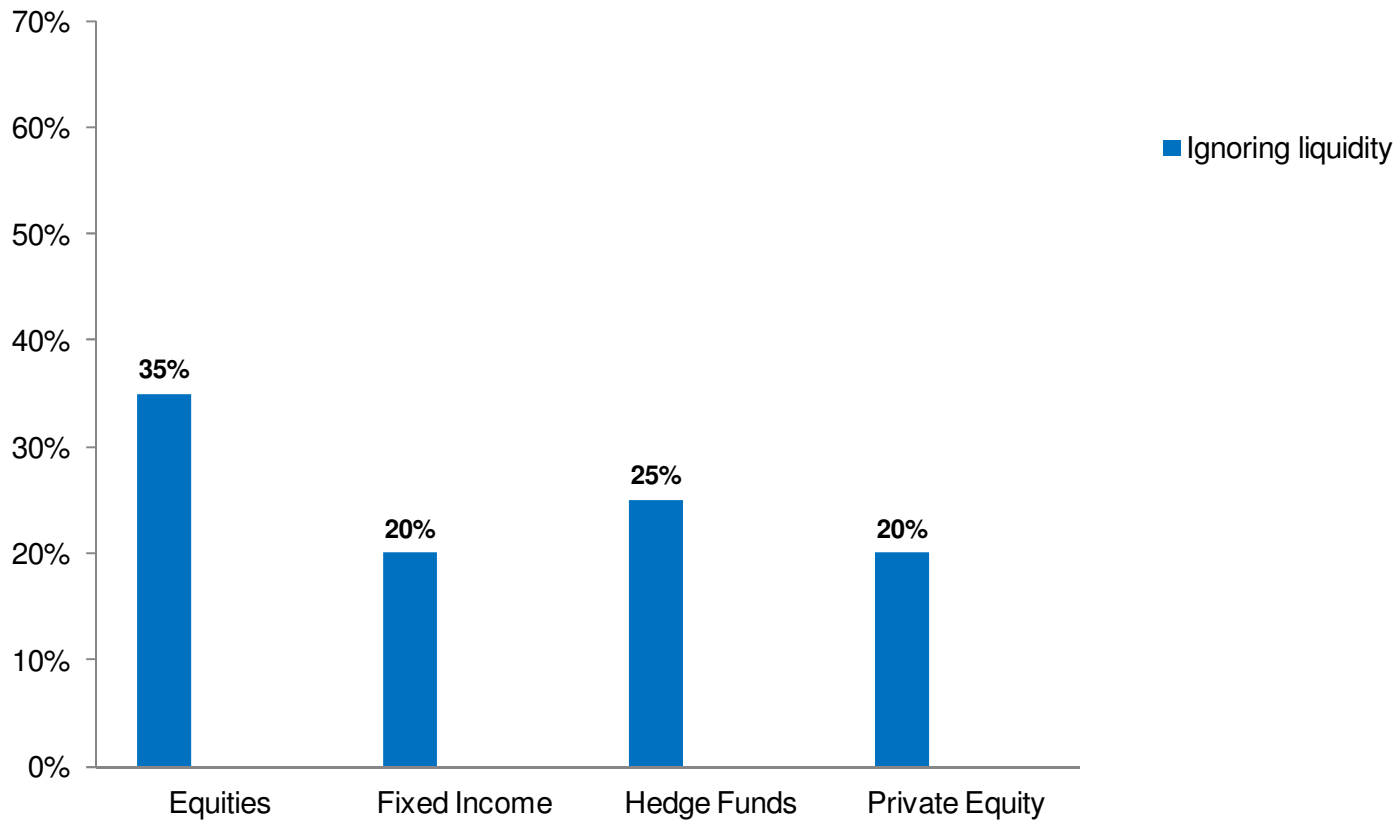
Liquidity benefits and illiquidity penalties			
	Return (bps)	**Risk (bps)**	**Attached to:**
Benefits			
Market timing	40	80	Liquid assets
Total shadow asset	**40**	**80**	**Liquid assets**
Penalties			
Sub-optimality cost from asset drift	46	0	Illiquid assets
Sub-optimality cost from capital calls	3	0	Illiquid assets
Borrowing cost from capital calls	20	16	Illiquid assets
Total shadow liability	**69**	**16**	**Illiquid assets**

Optimization inputs including shadow assets and liabilities

Asset class expected returns, risk, and correlations								
Asset Class	**Expected Return* (%)**	**Standard Deviation (%)**	**Correlations**					
			Equities	**Fixed**	**Hedge Funds**	**Private Equity**	**Shadow Asset**	**Shadow Liability**
Equities	9.4	15.0	1.00					
Fixed Income	4.0	8.0	0.25	1.00				
Hedge Funds	7.0	15.0	0.25	0.25	1.00			
Private Equity	15.8	30.0	0.75	0.00	0.25	1.00		
Shadow Asset	0.40	0.80	0.00	0.00	0.00	0.00	1.00	
Shadow Liability	-0.69	0.16	0.00	0.00	0.00	0.00	0.00	1.00

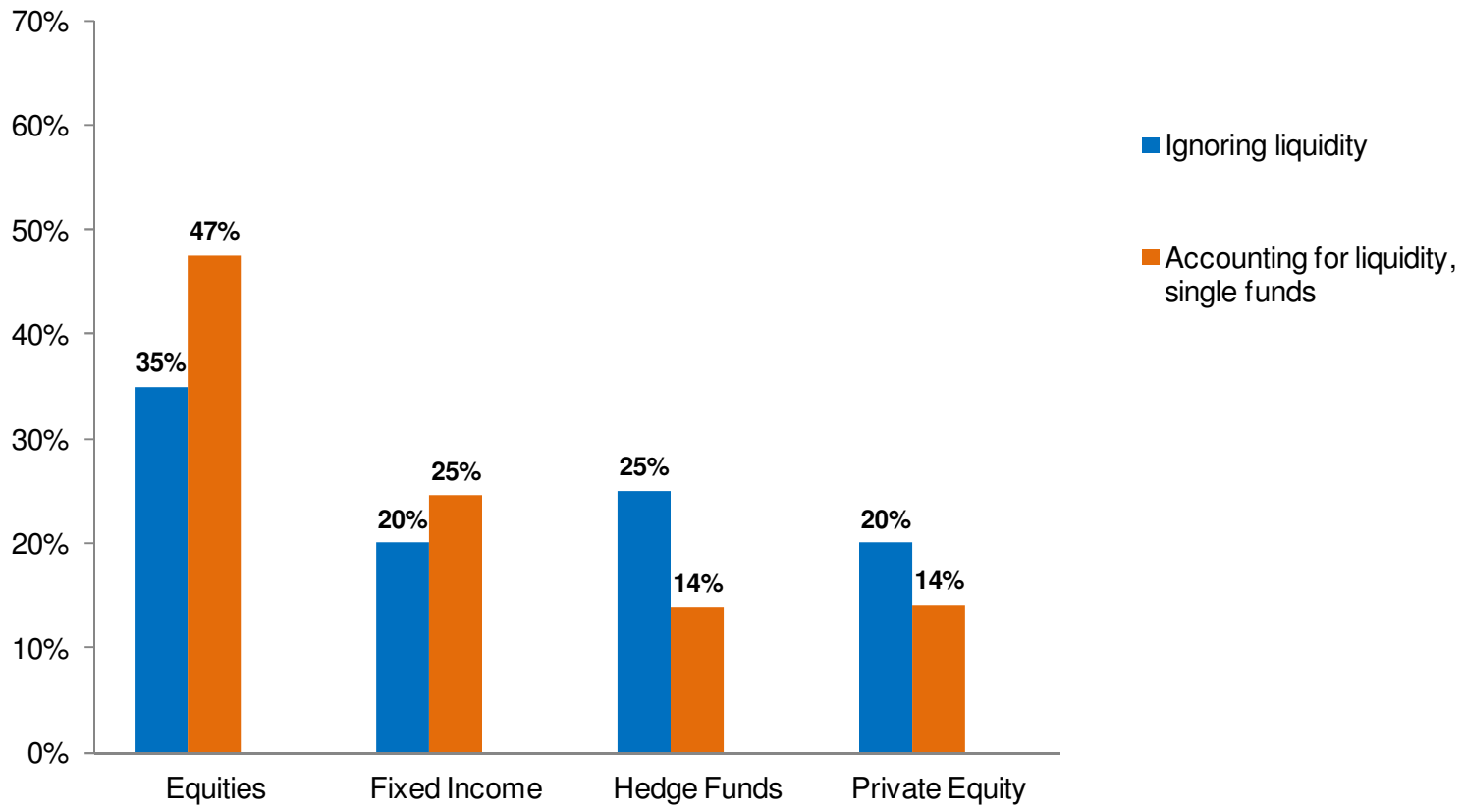
* Expected returns for equities, fixed income, hedge funds, and private equity are implied assuming risk aversion of 2.

Optimal allocations



* Assuming risk aversion of 2.

Optimal allocations



* Assuming risk aversion of 2.

Refinements

This case study, although rooted in the real world, represents a first pass at implementation, and thus relies on several simplifying assumptions.

- We assume that the shadow asset and liability are uncorrelated with the explicit assets of the portfolio.
- We also assume that explicit assets and shadow allocations are normally distributed.
- We model both absolute and partial illiquidity as though they are constant across assets and through time.
- Finally, we assume that the effects of liquidity are additive.

We could enrich our analysis by relaxing these assumptions.

Summary

- We propose that investors treat liquidity as shadow allocations.
  - If liquidity is deployed to increase a portfolio's initial expected utility, we attach a shadow asset to tradable assets.
  - If liquidity is deployed to prevent a decrease in expected utility, we attach a shadow liability to assets that are not tradable.
- The expected return and risk of these shadow allocations depend on how a particular investor uses liquidity.
- Before introducing the shadow asset and liability to a portfolio, we must first offset the effect of performance fees and fair value pricing on the illiquid asset's observed return and risk.

Summary

This approach improves upon other methods for incorporating liquidity into portfolio choice in four fundamental ways:

1. It mirrors what actually occurs within a portfolio.
2. It maps units of liquidity onto units of return and risk, so that investors can analyze liquidity within the same context as other portfolio decisions.
3. It distinguishes absolute illiquidity from partial illiquidity and enables investors to treat these attributes within a single unifying framework.
4. It recognizes that liquidity serves not only to meet demands for capital, but to exploit trading opportunities as well, thereby revealing that investors bear an illiquidity cost to the extent any fraction of a portfolio is immobile.

Thank you

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